



Integrated Research Centre (IREC)

Example Manual

Shear of RC beam – EN 1992-1-1



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Shear of RC beam according to EN 1992-1-1

1. Introduction

This template covers the calculation of shear strength of reinforced concrete (RC) beams according to EN-1992-1-1 [1]. Shear force caused by the load can be transferred via concrete if $V < V_{R,c}$ – Shear strength for RC beams without shear reinforcement (stirrups) – or via transversal reinforcement (stirrups)/concrete struts if $V > V_{R,c}$ – Shear strength for RC beams with shear reinforcement (stirrups). Although, if shear force is transferred via concrete, minimal shear reinforcement is required according to [1].

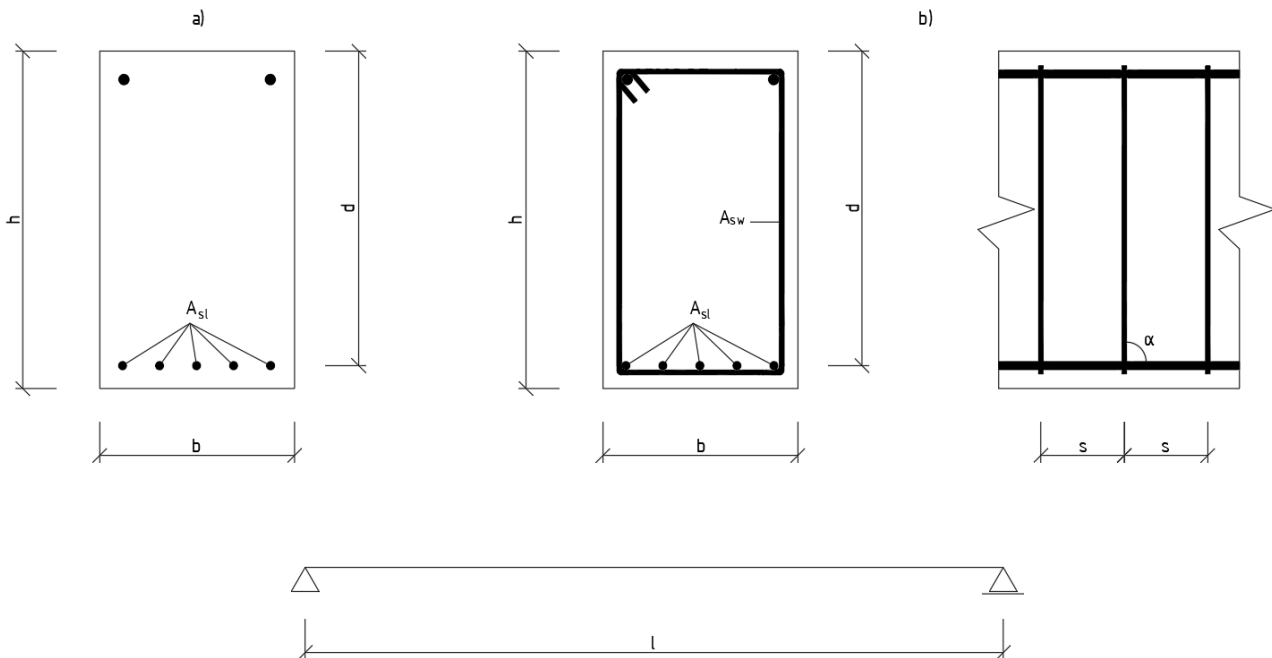
The rectangular RC beam of a single-span is simply supported with the span of $l = 5.0$ m, height of $h = 0.4$ m and width of $b = 0.15$ m. Cross-sectional area of concrete is $A_c = 0.06$ m². Axial force is not applied on beam, $N = 0$ kN. Beam is reinforced by 5 longitudinal bars of diameter of Ø12 mm, cross-sectional area of reinforcement (anchored to minimum anchor length) is $A_{sl} = 5.65 \cdot 10^{-4}$ m² and effective depth is $d = 0.379$ m (corresponds to a cover of 15 mm).

C25/30 concrete class is used with mean value of concrete compressive strength $f_{cm} = 31$ MPa.

In the case of shear strength with shear reinforcement, B500B steel is used, mean value of yield strength of shear reinforcement is $f_{ywm} = 510$ MPa. Beam is reinforced by two-legged stirrups with diameter of Ø8 mm and spacing $s = 0.2$ m. Cross-sectional area of stirrups is $A_{sw} = 1.01 \cdot 10^{-4}$ m². The angle between stirrups and longitudinal centroidal axis of stirrups is $\alpha = 90^\circ$. Cotangent of angle between the concrete compression strut and the beam axis perpendicular to the shear force should be between 1.0 and 2.5 (according to [1]), for this example it is considered to be $\cot \theta = 1.5$.

Recommended value of model uncertainty for deterministic calculation is $\Theta = 1.0$ as well as all partial safety factors.

For the reinforcement of a beam see Figure below: a) Reinforcement of a beam without stirrups, and b) Reinforcement of a beam with stirrups.



2. Input variables

The last column of the table contains the Excel Sheet numbers in which the given variable is entered and for which it is necessary to correctly define the individual input parameters in order to calculate:

1 – shear strength for a beam without stirrups,

2 – shear strength for a beam with stirrups.

Notation	Variable	Value	Unit	Model
b	Cross-sectional width (minimal width in tensile area)	0.150	m	1,2
d	Effective depth	0.379	m	1,2
A_c	Cross-sectional area of concrete	0.06	m ²	1
A_{sl}	Cross-sectional area of longitudinal reinforcement	$5.65 \cdot 10^{-4}$	m ²	1
f_{cm}	Mean value of concrete cylinder compressive strength	31	MPa	1,2
N	Applied axial force (compression positive)	0	kN	1
f_{ywm}	Mean value of yield strength of shear reinforcement	510	MPa	2
A_{sw}	Cross-sectional area of shear reinforcement	$1.01 \cdot 10^{-4}$	m ²	2
s	Spacing of the stirrups	0.2	m	2
α	Angle between stirrups and longitudinal centroidal axis	90	°	2
$\cot \theta$	Cotangent of angle between the concrete compression strut and the beam axis perpendicular to the shear force	1.5	-	2
θ	Uncertainty factor of model	1.00	-	1,2

3. Deterministic Calculation of Shear Strength for RC Beams without Stirrups

The calculation of shear strength of RC beam without shear reinforcement is calculated based on the formulas below.

Coefficient for concrete shear resistance $C_{R,c}$ is assumed to be:

$$C_{R,c} = \frac{0.18}{\gamma_c} = \frac{0.18}{1.0} = 0.18$$

A size effect factor k and longitudinal reinforcement ratio ρ_l are defined as:

$$k = \min \left(1 + \sqrt{\frac{200}{d}}; 2.0 \right) = \min \left(1 + \sqrt{\frac{200}{379}}; 2.0 \right) = \min(1.726; 2.0) = 1.726$$

$$\rho_l = \min \left(\frac{A_{sl}}{b \cdot d}; 2.0 \right) = \min \left(\frac{5.65 \cdot 10^{-4}}{0.15 \cdot 0.379}; 0.02 \right) = \min(0.01; 0.02) = 0.00994$$

Mean value of axial stress in the cross-sectional area due to loading σ_{cp} (compression is considered as positive $N > 0$ kN) is prescribed as:

$$\sigma_{cp} = \min \left(\frac{N}{A_c}; 0.2 f_{cm} \right) = \min \left(\frac{0}{0.06}; 0.2 \cdot 31 \right) = \min(0; 6.2) = 0 \text{ MPa}$$

Finally, shear strength $V_{R,c1}$ for RC beams without shear reinforcement is calculated as (considering $k_1 = 0.15$ [1]):

$$\begin{aligned} V_{R,c1} &= \theta \cdot \left[C_{R,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{cm})^{\frac{1}{3}} + k_1 \cdot \sigma_{cp} \right] \cdot d \cdot b \cdot 1000 \\ &= 1.00 \cdot \left[0.18 \cdot 1.726 \cdot (100 \cdot 0.00994 \cdot 31)^{\frac{1}{3}} + 0.15 \cdot 0 \right] \cdot 0.379 \cdot 0.15 \cdot 1000 = 55.38 \text{ kN} \end{aligned}$$

Minimum shear stress and minimum value of the shear strength are defined as:

$$v_{\min} = 0.035 \cdot k^{\frac{3}{2}} \cdot f_{\text{cm}}^{\frac{1}{2}} = 0.035 \cdot 1.726^{\frac{3}{2}} \cdot 31^{\frac{1}{2}} = 0.442 \text{ MPa}$$

$$V_{\text{R,c,min}} = \theta \cdot (v_{\min} + k_1 \cdot \sigma_{\text{cp}}) \cdot d \cdot b \cdot 1000 = 1.0 \cdot (0.442 + 0.15 \cdot 0) \cdot 0.379 \cdot 0.15 \cdot 1000 = 25.13 \text{ kN}$$

Shear strength $V_{\text{R,c}}$ for RC beams without stirrups is then:

$$V_{\text{R,c}} = \max(V_{\text{R,c1}}, V_{\text{R,c,min}}) = \max(55.38; 25.13) = 55.38 \text{ kN}$$

If shear strength $V_{\text{R,c}}$ is smaller than the shear force V , shear reinforcement must be designed.

4. Deterministic Calculation of Shear Strength for RC Beams with Stirrups

The calculation of shear strength of RC beam with shear reinforcement is calculated in a similar way, see the following text.

Lever arm of internal forces z (in the case that beam is not loaded by axial forces) can be expressed as:

$$z = 0.9 \cdot d = 0.9 \cdot 0.379 = 0.341 \text{ m}$$

Shear strength of shear reinforcement $V_{\text{R,s}}$ is:

$$\begin{aligned} V_{\text{R,s}} &= \theta \cdot \frac{A_{\text{sw}}}{s} \cdot z \cdot f_{\text{yw}} \cdot (\cot \theta + \cot \alpha) \cdot \sin \alpha \cdot 1000 = \\ &= 1.00 \cdot \frac{1.01 \cdot 10^{-4}}{0.2} \cdot 0.341 \cdot 510 \cdot (1.5 + \cot 90^\circ) \cdot \sin 90^\circ \cdot 1000 = 131.78 \text{ kN} \end{aligned}$$

Reduction factor for concrete strength at shear failure v depend on concrete compressive strength (in this case on mean value):

$$\begin{aligned} v &= 0.6 \quad \text{if} \quad f_{\text{cm}} \leq 60 \text{ MPa} \quad \text{or} \quad v = 0.6 \cdot \left(1 - \frac{f_{\text{cm}}}{250}\right) \quad \text{if} \quad f_{\text{cm}} > 60 \text{ MPa} \\ v &= 0.6 \quad \text{for} \quad f_{\text{cm}} = 31 \text{ MPa} \end{aligned}$$

Maximal shear strength of concrete compression diagonal $V_{\text{R,max}}$ is (considering $\alpha_{\text{cw}} = 1.0$):

$$\begin{aligned} V_{\text{R,max}} &= \theta \cdot \alpha_{\text{cw}} \cdot b \cdot z \cdot v \cdot f_{\text{cm}} \cdot \frac{\cot \theta + \cot \alpha}{1 + \cot^2 \theta} \cdot 1000 = \\ &= 1.00 \cdot 1.0 \cdot 0.15 \cdot 0.341 \cdot 0.6 \cdot 31 \cdot \frac{1.5 + \cot 90^\circ}{1 + 1.5^2} \cdot 1000 = 439.23 \text{ kN} \end{aligned}$$

Finally, shear strength for RC beams with stirrups is:

$$V_{\text{R,c}} = \min(V_{\text{R,max}}, V_{\text{R,s}}) = \min(439.23; 131.78) = 131.78 \text{ kN}$$

5. Literature

- [1] EN 1992-1-1:2004+A1:2014. Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings. Brussels: European Committee for Standardization (CEN), 2014.