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Co-funded by the European Union

Integrated Research Centre (IREC)

Example Manual

Bending and Deflection of RC slabs

IREC
ATCZ00068

BOKU University & Brno University of Technology
Version 2026-01

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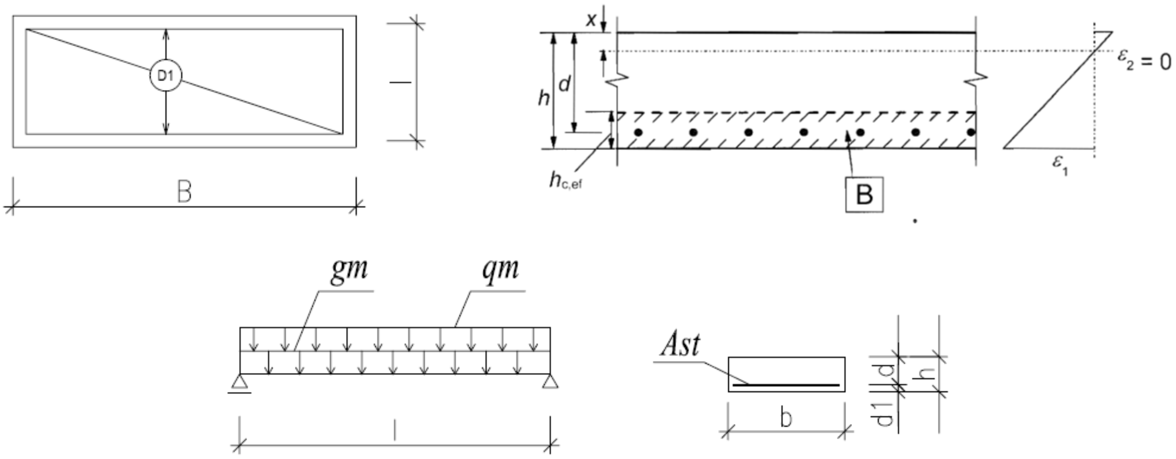
Bending and deflection of RC slab according to Eurocode 2

1. Problem Description

Based on the Eurocode 2 [1], this template covers the calculation of moment of resistance and deflection from loading, as well as the effect of creep and shrinkage of reinforced concrete slab D1 of B and l dimensions. The ceiling slab of the storage area (category of buildings E; see [2]) is considered to be simply supported with a span of l = 3.84 m, a height of h = 0.14 m, and loaded with a uniform continuous load: the mean value of the permanent load g_m = 5.40 kN/m (consisting of the dead load of the slab g_0 = 0.14·25 = 3.50 kN/m and other permanent loads of g_i = 1.90 kN/m) and the variable load q_m = 1.25 kN/m. The width of a cross-section for the calculation is assumed to be b = 1 m. Reinforcement with a reinforcing bar diameter of Ø12 mm and a spacing of 175 mm is designed, concrete cover c = 25 mm.

C25/30 concrete with characteristic and mean values of cubic compressive strength of f_{ck} = 25 MPa and f_{cm} = 33 MPa, mean concrete tensile strength f_{ctm} = 2.6 MPa, and B500B reinforcing steel with modulus of elasticity E_s = 200 GPa, mean value of yield strength f_{ym} = 525 MPa are used.

The moment of resistance and deflections are calculated at the structural design life, i.e. the age of concrete at the moment considered is assumed t = 50 years, the age of concrete at loading for the calculation of creep and shrinkage are assumed to be $t_{0,c}$ = 28 days for creep and $t_{0,s}$ = 7 days for shrinkage. Curing time of concrete was t_s = 4 days. Relative humidity of the ambient environment is RH = 40%, coefficients of cement type are α_{ds1} = 4 and α_{ds2} = 0.12 (based on [1] for cement Class N). Recommended value of model uncertainty for deterministic calculation is Θ = 1.0.



As an input parameter, the cross-sectional area of reinforcement is needed to be assessed. For b = 1 m the area of reinforcement is calculated as:

$$A_s = \frac{b}{(\text{spacing of reinforcement bars})} \cdot \pi \cdot \frac{Ø^2}{4} = \frac{1.00}{0.175} \cdot \pi \cdot \frac{0.012^2}{4} = 6.463 \cdot 10^{-4}\ \text{m}^2$$

2. Input variables

The last column of the table contains the Excel Sheet numbers in which the given variable is entered and for which it is necessary to correctly define the individual input parameters in order to calculate:
1 – moment of resistance,

2 – deflection due to load,

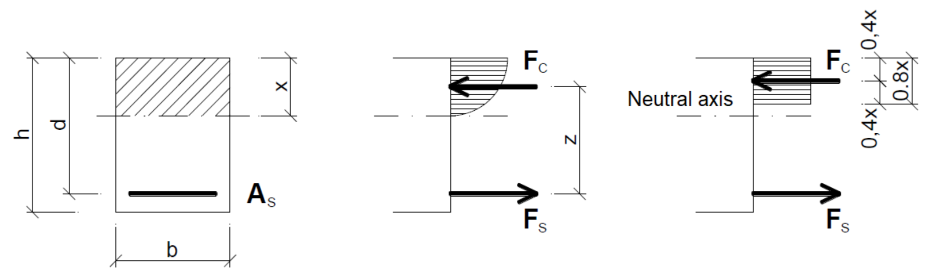
3 – deflection due to creep,

4 – deflection due to shrinkage.

Notation	Variable	Value	Unit	Model
h	Height of a cross-section	0.140	m	1,2,3,4
b	Width of a cross-section	1.000	m	1,2,3,4
l	Span	3.840	m	2,3,4
g_{m}	Permanent load	5.400	kN/m	2,3,4
q_{m}	Variable load	1.250	kN/m	2,3,4
Ψ_i	Coefficient for variable loads (i=1: frequent or i=2: quasi-static combination)	0.800	-	3,4
$Ø$	Diameter of a reinforcing bar	0.012	m	1,2,3,4
A_{s}	Cross sectional area of reinforcement (within the b width of a cross-section)	$6.463 \cdot 10^{-4}$	m²	1,2,3,4
c	Concrete cover	0.025	m	1,2,3,4
f_{ym}	Mean yield strength of reinforcement	525.000	MPa	1
E_{s}	Modulus of elasticity of reinforcing steel	200.000	GPa	2,3,4
f_{ck}	Characteristic value of concrete cylinder compressive strength	25.000	MPa	1,4
f_{cm}	Mean value of concrete cylinder compressive strength	33.000	MPa	1,2,3,4
f_{ctm}	Mean value of axial tensile strength of concrete	2.600	MPa	2,3,4
t	Age of concrete at the moment considered (e.g. design life of the structure)	50.000	years	3,4
$t_{0,\mathrm{c}}$	Age of concrete at loading for the calculation of deflection due to creep	28.000	days	3
$t_{0,\mathrm{s}}$	Age of concrete at loading for the calculation of deflection due to shrinkage	7.000	days	4
t_{s}	Age of concrete at the start of drying (usually the end of concrete curing)	4.000	days	4
RH	The relative humidity of the ambient environment	40.000	%	3,4
α_{ds1}	Coefficient which depends on the type of cement	4.000	-	4
α_{ds2}	Coefficient which depends on the type of cement	0.120	-	4
Θ	Uncertainty factor of model	1.000	-	1,2,3,4

3. Deterministic Calculation of Moment of Resistance

This section describes the calculation of the moment of resistance of a simply supported plate loaded with a uniform continuous load as specified above. The Excel Template sheet is labelled 'Moment_resistance'.



The calculation is pictured in the figure and can be commented as follows. At first, the effective depth of a section is calculated as:

$$d = h - c - \frac{Ø}{2} = 0.140 - 0.025 - \frac{0.012}{2} = 0.109 \text{ m}$$

Neutral axis depth is assessed based on the formula:

$$x = \frac{A_s f_{ym}}{\lambda b f_{cm}} = \frac{6.463 \cdot 10^{-4} \cdot 525}{0.8 \cdot 1 \cdot 33} = 0.0129 \text{ m}$$

with the coefficient λ = 0.8 for the equivalent height of the pressure area. For cases with f_{ck} > 50 MPa, the coefficient is calculated according to:

$$\lambda = 0.8 - \frac{f_{ck} - 50}{400}$$

Tensile force of reinforcement:

$$F_s = A_s f_{ym} = 6.463 \cdot 10^{-4} \cdot 525 \cdot 10^3 = 339.31 \text{ kN}$$

Finally, the moment of resistance is calculated as:

$$\boldsymbol{M_R = \theta F_s(d - 0.5\lambda x) = 1.00 \cdot 339.31 \cdot (0.109 - 0.5 \cdot 0.8 \cdot 0.0129) = 35.23 \text{ kNm}}$$

4. Deterministic Calculation of Deflection Caused by Bending

This section describes the calculation of a simply supported plate loaded with a uniform continuous load as specified above. The Excel Template sheet is labelled 'Deflection_bending'. The steps of the calculation are as follows:

A secant modulus of elasticity for concrete as:

$$E_{\text{cm}} = 22 \cdot \left(\frac{f_{\text{cm}}}{10}\right)^{0.3} = 22 \cdot \left(\frac{33}{10}\right)^{0.3} = 31.476 \text{ GPa}$$

An effective modulus of elasticity of concrete:

$$E_{\text{c,eff}} = \frac{E_{\text{cm}}}{1 + \varphi(t, t_0)} = \frac{31.476}{1 + 3.056} = 7.760 \text{ GPa} \quad \text{(creep)}$$

$$E_{\text{c,eff}} = \frac{E_{\text{cm}}}{1 + \varphi(t, t_0)} = \frac{31.476}{1 + 3.971} = 6.332 \text{ GPa} \quad \text{(shrinkage)}$$

The effective depth of a section is calculated as:

$$d = h - c - \frac{\text{Ø}}{2} = 0.140 - 0.025 - \frac{0.012}{2} = 0.109 \text{ m}$$

Ratio of modulus of elasticity:

$$n = \frac{E_s}{E_{\text{cm}}} = \frac{200}{31.476} = 6.354$$

Neutral axis depth:

$$x = \frac{n}{b} \cdot A_s \cdot \left(-1 + \sqrt{1 + \frac{2 \cdot b}{n} \cdot \frac{d}{A_s}}\right) = \frac{6.354}{1.00} \cdot 6.463 \cdot 10^{-4} \cdot \left(-1 + \sqrt{1 + \frac{2 \cdot 1.00}{6.354} \cdot \frac{0.109}{6.463 \cdot 10^{-4}}}\right) = 0.0261 \text{ m}$$

Area of an ideal cross-section A_{I} and area of ideal cross-section damaged by cracks A_{II}:

$$A_{\text{I}} = bh + nA_s = 1.00 \cdot 0.14 + 6.354 \cdot 6.463 \cdot 10^{-4} = 0.1441 \text{ m}^2$$

$$A_{\text{II}} = bx + nA_s = 1.00 \cdot 0.0261 + 6.354 \cdot 6.463 \cdot 10^{-4} = 0.0302 \text{ m}^2$$

Distance of the centre of gravity of the ideal cross-section from the upper edge:

$$a_{\text{Cg,I}} = \frac{b \cdot h \cdot \frac{h}{2} + n \cdot A_s \cdot d}{A_{\text{I}}} = \frac{1.00 \cdot 0.14 \cdot \frac{0.14}{2} + 6.354 \cdot 6.463 \cdot 10^{-4} \cdot 0.109}{0.1441} = 0.0711 \text{ m}$$

For the cross-section damaged by cracks, it becomes to:

$$a_{\mathrm{Cg,II}} = \frac{b \cdot x \cdot \frac{x}{2} + n \cdot A_s \cdot d}{A_{\mathrm{II}}} = \frac{1.00 \cdot 0.0261 \cdot \frac{0.0261}{2} + 6.354 \cdot 6.463 \cdot 10^{-4} \cdot 0.109}{0.0302} = 0.0261\ \mathrm{m}$$

Moment of inertia of the ideal cross-section to its centre of gravity:

$$I_{\mathrm{I}} = \frac{1}{12} bh^3 + bh\left(a_{\mathrm{Cg,I}} - \frac{h}{2}\right)^2 + nA_s\left(d - a_{\mathrm{Cg,I}}\right)^2$$
$$= \frac{1}{12} \cdot 1.00 \cdot 0.14^3 + (1.00 \cdot 0.14) \cdot \left(0.0711 - \frac{0.14}{2}\right)^2 + 6.354 \cdot 6.463 \cdot 10^{-4} \cdot (0.109 - 0.0711)^2$$
$$= 2.347 \cdot 10^{-4}\ \mathrm{m}^4$$

For the cross-section damaged by cracks, it becomes to:

$$I_{\mathrm{II}} = \frac{1}{12} bx^3 + bx\left(a_{\mathrm{Cg,II}} - \frac{x}{2}\right)^2 + nA_s\left(d - a_{\mathrm{Cg,II}}\right)^2$$
$$= \frac{1}{12} \cdot 1.00 \cdot 0.0261^3 + (1.00 \cdot 0.0261) \cdot \left(0.0261 - \frac{0.0261}{2}\right)^2 + 6.354 \cdot 6.463 \cdot 10^{-4}$$
$$\cdot (0.109 - 0.0261)^2 = 3.415 \cdot 10^{-5}\ \mathrm{m}^4$$

Maximum bending moment at the centre of the beam caused by permanent load and variable load (for a characteristic combination of load):

$$M_{\mathrm{sk}} = \frac{1}{8} \cdot (g_m + q_m) \cdot l^2 = \frac{1}{8} \cdot (5.40 + 1.25) \cdot 3.84^2 = 12.257\ \mathrm{kNm}$$

Bending moment at crack initiation:

$$M_{\mathrm{cr}} = f_{\mathrm{ctm}} \cdot \frac{I_{\mathrm{I}}}{h - a_{\mathrm{Cg,I}}} = 2.600 \cdot 10^3 \cdot \frac{2.347 \cdot 10^{-4}}{0.14 - 0.0711} = 8.859\ \mathrm{kNm}$$

Distribution coefficient allowing for tensioning stiffening at a section is calculated as:

$$\xi = 1 - \beta_{\mathrm{l}} \cdot \left(\frac{M_{\mathrm{cr}}}{M_{\mathrm{sk}}}\right)^2 = 1 - 1.0 \cdot \left(\frac{8.859}{12.257}\right)^2 = 0.478$$

with β_{l} being the coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain. For the calculation of deflection due to loading, $\beta_{\mathrm{l}} = 1.0$ as defined for short-term load.

The stiffness of the cross-section in individual stages is defined for stage m1 – intact cross-section and for stage m2 – cross-section damaged by cracks as:

$$f_{\mathrm{m,1}} = (1 - \xi) \cdot \frac{1}{E_{\mathrm{cm}} \cdot I_{\mathrm{I}}} = (1 - 0.478) \cdot \frac{1}{31.476 \cdot 10^6 \cdot 2.347 \cdot 10^{-4}} = 7.071 \cdot 10^{-5}\ \mathrm{kN^{-1}m^{-2}}$$

$$f_{\mathrm{m,2}} = \xi \cdot \frac{1}{E_{\mathrm{cm}} \cdot I_{\mathrm{II}}} = 0.478 \cdot \frac{1}{31.476 \cdot 10^6 \cdot 3.415 \cdot 10^{-5}} = 4.443 \cdot 10^{-4}\ \mathrm{kN^{-1}m^{-2}}$$

Finally, deflection due to loading (uniform continuous load in this case) is:

$$\boldsymbol{w = \theta\left[\frac{5}{48} M_{\mathrm{sk}}(f_{\mathrm{m,1}} + f_{\mathrm{m,2}}) l^2\right] = 1.000 \cdot \left[\frac{5}{48} \cdot 12.257 \cdot (7.071 \cdot 10^{-5} + 4.443 \cdot 10^{-4}) \cdot 3.84^2\right]}$$
$$\boldsymbol{= 9.696 \cdot 10^{-3}\ \mathrm{m}}$$

5. Deterministic Calculation of Deflections due to Creep and Shrinkage

This section describes the calculation of deflections in a simply supported plate due to creep and shrinkage. Since most coefficients are calculated in the same way for both creep and shrinkage, the calculations for these two phenomena are described together. The individual sheets of the Excel Template are labelled 'Defelection_creep' and 'Defelection_shrinkage, respectively.

A perimeter of the member in contact with the atmosphere (tom and bottom slab surface for this case):

$$u = 2b = 2 \cdot 1000 = 2000 \text{ mm}$$

Notional member size:

$$h_0 = \frac{2A_c}{u} = \frac{2bh}{u} = \frac{2 \cdot 1000 \cdot 140}{2000} = 140 \text{ mm}$$

Coefficients to consider the influence of the concrete strength:

$$\alpha_1 = \left(\frac{35}{f_{\text{cm}}}\right)^{0.7} = \left(\frac{35}{33}\right)^{0.7} = 1.042$$

$$\alpha_2 = \left(\frac{35}{f_{\text{cm}}}\right)^{0.2} = \left(\frac{35}{33}\right)^{0.2} = 1.012$$

$$\alpha_1 = \left(\frac{35}{f_{\text{cm}}}\right)^{0.5} = \left(\frac{35}{33}\right)^{0.5} = 1.030$$

Coefficient depending on the relative humidity (*RH* in %) and the notional member size (h_0 in mm) is:

$$\beta_{\text{H}} = 1.5 \cdot [1 + (0.012 \cdot RH)^{18}] \cdot h_0 + 250 \leq \beta_{\text{H,lim}} \qquad \text{for } f_{\text{cm}} \leq 35 \text{ MPa} \quad \text{(a)}$$

$$\beta_{\text{H}} = 1.5 \cdot [1 + (0.012 \cdot RH)^{18}] \cdot h_0 + 250 \cdot \alpha_3 \leq \beta_{\text{H,lim}} \cdot \alpha_3 \qquad \text{for } f_{\text{cm}} > 35 \text{ MPa} \quad \text{(b)}$$

with $\beta_{\text{H,lim}} = 1500$. For the input values and $f_{\text{cm}} \leq 35$ MPa:

$$\beta_{\text{H}} = 1.5 \cdot [1 + (0.012 \cdot RH)^{18}] \cdot h_0 + 250 \leq \beta_{\text{H,lim}}$$

$$\beta_{\text{H}} = 1.5 \cdot [1 + (0.012 \cdot 40)^{18}] \cdot 140 + 250 = 460 \quad \leq \beta_{\text{H,lim}} = 1500$$

A factor to allow for the effect of concrete age at loading on the notional creep coefficient is defined as:

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.20})} = \frac{1}{(0.1 + 28^{0.2})} = 0.488 \quad \text{(creep)}$$

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.20})} = \frac{1}{(0.1 + 7^{0.2})} = 0.635 \quad \text{(shrinkage)}$$

with $t_0 = t_{0,\text{c}} = 28$ days for creep, and $t_0 = t_{0,\text{s}} = 7$ days for shrinkage.

A factor to allow for the effect of concrete strength on the notional creep coefficient is:

$$\beta(f_{\text{cm}}) = \frac{16.8}{\sqrt{f_{\text{cm}}}} = \frac{16.8}{\sqrt{33}} = 2.925$$

A factor to allow for the effect of relative humidity on the notional creep coefficient:

$$\varphi_{\mathrm{RH}} = 1 + \frac{1 - \frac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} \qquad \text{for } f_{\mathrm{cm}} \leq 35 \text{ MPa} \quad \text{(a)}$$

$$\varphi_{\mathrm{RH}} = \left[1 + \frac{1 - \frac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1\right] \cdot \alpha_2 \qquad \text{for } f_{\mathrm{cm}} > 35 \text{ MPa} \quad \text{(b)}$$

For the case $f_{\mathrm{cm}} \leq 35$ MPa:

$$\varphi_{\mathrm{RH}} = 1 + \frac{1 - \frac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} = 1 + \frac{1 - \frac{40}{100}}{0.1 \cdot \sqrt[3]{140}} = 2.156$$

A coefficient to describe the development of creep with time after loading is calculated as:

$$\beta_{\mathrm{c}}(t, t_0) = \left[\frac{(t - t_0)}{(\beta_{\mathrm{H}} + t - t_0)}\right]^{0.3} = \left[\frac{(50 \cdot 365.25 - 28)}{(460.00 + 50 \cdot 365.25 - 28)}\right]^{0.3} = 0.993 \quad \text{(creep)}$$

$$\beta_{\mathrm{c}}(t, t_0) = \left[\frac{(t - t_0)}{(\beta_{\mathrm{H}} + t - t_0)}\right]^{0.3} = \left[\frac{(50 \cdot 365.25 - 7)}{(460.00 + 50 \cdot 365.25 - 7)}\right]^{0.3} = 0.993 \quad \text{(shrinkage)}$$

with $t_0 = t_{0,\mathrm{c}} = 28$ days for creep, and $t_0 = t_{0,\mathrm{s}} = 7$ days for shrinkage.

The notional creep coefficient:

$$\varphi_0 = \varphi_{\mathrm{RH}} \cdot \beta(f_{\mathrm{cm}}) \cdot \beta(t_0) = 2.156 \cdot 2.925 \cdot 0.488 = 3.079 \quad \text{(creep)}$$

$$\varphi_0 = \varphi_{\mathrm{RH}} \cdot \beta(f_{\mathrm{cm}}) \cdot \beta(t_0) = 2.156 \cdot 2.925 \cdot 0.635 = 4.000 \quad \text{(shrinkage)}$$

The creep coefficient is then calculated as:

$$\varphi(t, t_0) = \varphi_0 \cdot \beta_{\mathrm{c}}(t, t_0) = 3.079 \cdot 0.993 = 3.056 \quad \text{(creep)}$$

$$\varphi(t, t_0) = \varphi_0 \cdot \beta_{\mathrm{c}}(t, t_0) = 4.000 \cdot 0.993 = 3.971 \quad \text{(shrinkage)}$$

When determining deflection, it is also necessary to calculate a secant modulus of elasticity for concrete as:

$$E_{\mathrm{cm}} = 22 \cdot \left(\frac{f_{\mathrm{cm}}}{10}\right)^{0.3} = 22 \cdot \left(\frac{33}{10}\right)^{0.3} = 31.476 \text{ GPa}$$

An effective modulus of elasticity of concrete:

$$E_{\mathrm{c,eff}} = \frac{E_{\mathrm{cm}}}{1 + \varphi(t, t_0)} = \frac{31.476}{1 + 3.056} = 7.760 \text{ GPa} \quad \text{(creep)}$$

$$E_{\mathrm{c,eff}} = \frac{E_{\mathrm{cm}}}{1 + \varphi(t, t_0)} = \frac{31.476}{1 + 3.971} = 6.332 \text{ GPa} \quad \text{(shrinkage)}$$

The effective depth of a section is calculated as:

$$d = h - c - \frac{\varnothing}{2} = 0.140 - 0.025 - \frac{0.012}{2} = 0.109 \text{ m}$$

Ratio of modulus of elasticity:

$$n_{\text{eff}} = \frac{E_s}{E_{c,\text{eff}}} = \frac{200}{7.760} = 25.773 \quad \text{(creep)}$$

$$n_{\text{eff}} = \frac{E_s}{E_{c,\text{eff}}} = \frac{200}{6.332} = 31.584 \quad \text{(shrinkage)}$$

And neutral axis depth:

$$x = \frac{n_{\text{eff}}}{b} \cdot A_s \cdot \left(-1 + \sqrt{1 + \frac{2 \cdot b}{n_{\text{eff}}} \cdot \frac{d}{A_s}} \right) = \frac{25.773}{1.00} \cdot 6.463 \cdot 10^{-4} \cdot \left(-1 + \sqrt{1 + \frac{2 \cdot 1.00}{25.773} \cdot \frac{0.109}{6.463 \cdot 10^{-4}}} \right)$$
$$= 0.0459 \text{ m} \quad \text{(creep)}$$

$$x = \frac{n_{\text{eff}}}{b} \cdot A_s \cdot \left(-1 + \sqrt{1 + \frac{2 \cdot b}{n_{\text{eff}}} \cdot \frac{d}{A_s}} \right) = \frac{31.584}{1.00} \cdot 6.463 \cdot 10^{-4} \cdot \left(-1 + \sqrt{1 + \frac{2 \cdot 1.00}{31.584} \cdot \frac{0.109}{6.463 \cdot 10^{-4}}} \right)$$
$$= 0.0493 \text{ m} \quad \text{(shrinkage)}$$

Area of an ideal cross-section:

$$A_{\text{I}} = bh + n_{\text{eff}} A_s = 1.00 \cdot 0.14 + 25.773 \cdot 6.463 \cdot 10^{-4} = 0.1567 \text{ m}^2 \quad \text{(creep)}$$

$$A_{\text{I}} = bh + n_{\text{eff}} A_s = 1.00 \cdot 0.14 + 31.584 \cdot 6.463 \cdot 10^{-4} = 0.1604 \text{ m}^2 \quad \text{(shrinkage)}$$

Area of ideal cross-section damaged by cracks:

$$A_{\text{II}} = bx + n_{\text{eff}} A_s = 1.00 \cdot 0.0459 + 25.773 \cdot 6.463 \cdot 10^{-4} = 0.0625 \text{ m}^2 \quad \text{(creep)}$$

$$A_{\text{II}} = bx + n_{\text{eff}} A_s = 1.00 \cdot 0.0493 + 31.584 \cdot 6.463 \cdot 10^{-4} = 0.0698 \text{ m}^2 \quad \text{(shrinkage)}$$

Distance of the centre of gravity of the ideal cross-section from the upper edge:

$$a_{\text{Cg,I}} = \frac{b \cdot h \cdot \frac{h}{2} + n_{\text{eff}} \cdot A_s \cdot d}{A_{\text{I}}} = \frac{1.00 \cdot 0.14 \cdot \frac{0.14}{2} + 25.773 \cdot 6.463 \cdot 10^{-4} \cdot 0.109}{0.1567} = 0.0741 \text{ m} \quad \text{(creep)}$$

$$a_{\text{Cg,I}} = \frac{b \cdot h \cdot \frac{h}{2} + n_{\text{eff}} \cdot A_s \cdot d}{A_{\text{I}}} = \frac{1.00 \cdot 0.14 \cdot \frac{0.14}{2} + 31.584 \cdot 6.463 \cdot 10^{-4} \cdot 0.109}{0.1604} = 0.0750 \text{ m} \quad \text{(shrinkage)}$$

For the cross-section damaged by cracks, it becomes to:

$$a_{\text{Cg,II}} = \frac{b \cdot x \cdot \frac{x}{2} + n_{\text{eff}} \cdot A_s \cdot d}{A_{\text{II}}} = \frac{1.00 \cdot 0.0459 \cdot \frac{0.0459}{2} + 25.773 \cdot 6.463 \cdot 10^{-4} \cdot 0.109}{0.0625} = 0.0459 \text{ m} \quad \text{(creep)}$$

$$a_{\text{Cg,II}} = \frac{b \cdot x \cdot \frac{x}{2} + n_{\text{eff}} \cdot A_s \cdot d}{A_{\text{II}}} = \frac{1.00 \cdot 0.0493 \cdot \frac{0.0493}{2} + 31.584 \cdot 6.463 \cdot 10^{-4} \cdot 0.109}{0.0698} = 0.0493 \text{ m} \quad \text{(shrinkage)}$$

Next, the cross-sectional characteristics need to be calculated for the intact cross-section and the cross-section damaged by cracks. The moment of inertia of the ideal cross-section to its centre of gravity is calculated based on formula:

$$I_{\mathrm{I}} = \frac{1}{12}bh^3 + bh\left(a_{\mathrm{Cg,I}} - \frac{h}{2}\right)^2 + n_{\mathrm{eff}}A_s\left(d - a_{\mathrm{Cg,I}}\right)^2$$
$$= \frac{1}{12} \cdot 1.00 \cdot 0.14^3 + (1.00 \cdot 0.14) \cdot \left(0.0741 - \frac{0.14}{2}\right)^2 + 25.773 \cdot 6.463 \cdot 10^{-4} \cdot (0.109 - 0.0741)^2$$
$$= 2.513 \cdot 10^{-4}\ \mathrm{m}^4 \quad \text{(creep)}$$

$$I_{\mathrm{I}} = \frac{1}{12}bh^3 + bh\left(a_{\mathrm{Cg,I}} - \frac{h}{2}\right)^2 + n_{\mathrm{eff}}A_s\left(d - a_{\mathrm{Cg,I}}\right)^2$$
$$= \frac{1}{12} \cdot 1.00 \cdot 0.14^3 + (1.00 \cdot 0.14) \cdot \left(0.0750 - \frac{0.14}{2}\right)^2 + 31.584 \cdot 6.463 \cdot 10^{-4} \cdot (0.109 - 0.0750)^2$$
$$= 2.558 \cdot 10^{-4}\ \mathrm{m}^4 \quad \text{(shrinkage)}$$

For the cross-section damaged by cracks, it becomes to:

$$I_{\mathrm{II}} = \frac{1}{12}bx^3 + bx\left(a_{\mathrm{Cg,II}} - \frac{x}{2}\right)^2 + n_{\mathrm{eff}}A_s\left(d - a_{\mathrm{Cg,II}}\right)^2$$
$$= \frac{1}{12} \cdot 1.00 \cdot 0.0459^3 + (1.00 \cdot 0.0459) \cdot \left(0.0459 - \frac{0.0459}{2}\right)^2 + 25.773 \cdot 6.463 \cdot 10^{-4}$$
$$\cdot (0.109 - 0.0459)^2 = 9.856 \cdot 10^{-5}\ \mathrm{m}^4 \quad \text{(creep)}$$

$$I_{\mathrm{II}} = \frac{1}{12}bx^3 + bx\left(a_{\mathrm{Cg,II}} - \frac{x}{2}\right)^2 + n_{\mathrm{eff}}\left(d - a_{\mathrm{Cg,II}}\right)^2$$
$$= \frac{1}{12} \cdot 1.00 \cdot 0.0493^3 + (1.00 \cdot 0.0493) \cdot \left(0.0493 - \frac{0.0493}{2}\right)^2 + 31{,}584 \cdot 6.463 \cdot 10^{-4}$$
$$\cdot (0.109 - 0.0493)^2 = 1.127 \cdot 10^{-4}\ \mathrm{m}^4 \quad \text{(shrinkage)}$$

For the calculation of deflection due to shrinkage, the static moment of the cross-sectional area of the reinforcement relative to the centre of gravity of the cross-section has to be assessed for the intact section and for the section damaged by cracks:

$$S_{\mathrm{I}} = A_s\left(d - a_{\mathrm{Cg,I}}\right) = 6.463 \cdot 10^{-4} \cdot (0.109 - 0.0750) = 2.200 \cdot 10^{-5}\ \mathrm{m}^3 \quad \text{(shrinkage only)}$$

$$S_{\mathrm{II}} = A_s\left(d - a_{\mathrm{Cg,II}}\right) = 6.463 \cdot 10^{-4} \cdot (0.109 - 0.0493) = 3.855 \cdot 10^{-5}\ \mathrm{m}^3 \quad \text{(shrinkage only)}$$

Maximum bending moment at the centre of the beam caused by permanent load and variable load is calculated for both creep and shrinkage as:

$$M_{\mathrm{sq}} = \frac{1}{8} \cdot (g_m + \Psi_i \cdot q_m) \cdot l^2 = \frac{1}{8} \cdot (5.40 + 0.8 \cdot 1.25) \cdot 3.84^2 = 11.796\ \mathrm{kNm}$$

with $\Psi_i = \Psi_2 = 0.8$ for a quasi-static combination which is assumed for long-term loading.

Bending moment at crack initiation:

$$M_{\mathrm{cr}} = f_{\mathrm{ctm}} \cdot \frac{I_{\mathrm{I}}}{h - a_{\mathrm{Cg,I}}} = 2.600 \cdot 10^3 \cdot \frac{2.513 \cdot 10^{-4}}{0.14 - 0.0741} = 9.922\ \mathrm{kNm} \quad \text{(creep)}$$

$$M_{\mathrm{cr}} = f_{\mathrm{ctm}} \cdot \frac{I_{\mathrm{I}}}{h - a_{\mathrm{Cg,I}}} = 2.600 \cdot 10^3 \cdot \frac{2.558 \cdot 10^{-4}}{0.14 - 0.0750} = 10.225\ \mathrm{kNm} \quad \text{(shrinkage)}$$

Distribution coefficient allowing for tensioning stiffening at a section is calculated as:

$$\xi = 1 - \beta_1 \cdot \left(\frac{M_{\mathrm{cr}}}{M_{\mathrm{sq}}}\right)^2 = 1 - 0.5 \cdot \left(\frac{9.922}{11.796}\right)^2 = 0.646 \quad \text{(creep)}$$

$$\xi = 1 - \beta_l \cdot \left(\frac{M_{cr}}{M_{sq}}\right)^2 = 1 - 0.5 \cdot \left(\frac{10.225}{11.796}\right)^2 = 0.624 \quad \text{(shrinkage)}$$

with β_l being the coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain. For the case of creep and shrinkage, $\beta_l = 0.5$ as defined for short-term load.

For the calculation of deflection due to creep, the stiffness of the cross-section in individual stages is defined for stage m1 – intact cross-section and for stage m2 – cross-section damaged by cracks as:

$$f_{m,1} = (1 - \xi) \cdot \frac{1}{E_{c,eff} \cdot I_I} = (1 - 0.646) \cdot \frac{1}{7.760 \cdot 10^6 \cdot 2.513 \cdot 10^{-4}} = 1.814 \cdot 10^{-4}\ \mathrm{kN^{-1}m^{-2}}$$

$$f_{m,2} = \xi \cdot \frac{1}{E_{c,effm} \cdot I_{II}} = 0.646 \cdot \frac{1}{7.760 \cdot 10^6 \cdot 9.856 \cdot 10^{-5}} = 8.450 \cdot 10^{-4}\ \mathrm{kN^{-1}m^{-2}}$$

Finally, deflection due to creep is:

$$\boldsymbol{w_{creep} = \theta\left[\frac{5}{48} M_{Sq}(f_{m,1} + f_{m,2}) l^2\right] = 1.000 \cdot \left[\frac{5}{48} \cdot 11.796 \cdot (1.814 \cdot 10^{-4} + 8.450 \cdot 10^{-4}) \cdot 3.84^2\right] = 18.598 \cdot 10^{-3}\ \mathrm{m}}$$

To calculate deflection due to shrinkage, it is necessary to calculate additional parameters. The influence of environmental relative humidity is assumed as:

$$\beta_{RH} = 1.55[1 - (RH/100)^3] = 1.55[1 - (40/100)^3] = 1.451$$

Nominal value of proportional shrinkage strain due to drying of concrete:

$$\varepsilon_{cd,0} = 0.85\left[(220 + 110\alpha_{ds1}) \cdot \exp\left(-\alpha_{ds2}\frac{f_{cm}}{10}\right)\right] \cdot 10^{-6} \cdot \beta_{RH} = 0.85\left[(220 + 110 \cdot 4.00) \cdot \exp\left(-0.12 \cdot \frac{33}{10}\right)\right] \cdot 10^{-6} \cdot 1.451 = 5.478 \cdot 10^{-4}$$

Final value of proportional shrinkage strain caused by concrete drying:

$$\varepsilon_{cd,\infty} = k_h \cdot \varepsilon_{cd,0} = 0.940 \cdot 5.478 \cdot 10^{-4} = 5.149 \cdot 10^{-4}$$

with coefficient dependent on nominal dimension $k_h = 0.940$ for $h_0 = 140$ mm according to following table; linear interpolation is used to determine intermediate values.

h_0 [mm]	k_h [–]
100	1.00
200	0.85
300	0.75
≥ 500	0.70

Coefficient of time development of drying shrinkage is assumed as:

$$\beta_{ds}(t,\ t_s) = \frac{t - t_s}{(t - t_s) + 0.04\sqrt{h_0^3}} = \frac{50 \cdot 365.25 - 4}{(50 \cdot 365.25 - 4) + 0.04\sqrt{140^3}} = 0.996$$

Proportional shrinkage strain due to drying over time t:

$$\varepsilon_{cd}(t) = \beta_{ds}(t,\ t_s) \cdot \varepsilon_{cd,\infty} = 0.996 \cdot 5.149 \cdot 10^{-4} = 5.130 \cdot 10^{-4}$$

Final value of autogenous shrinkage strain:

$$\varepsilon_{\mathrm{ca},\infty} = 2.5(f_{\mathrm{ck}} - 10) \cdot 10^{-6} = 2.5(25 - 10) \cdot 10^{-6} = 3.750 \cdot 10^{-5}$$

Coefficient of time development of autogenous shrinkage:

$$\beta_{\mathrm{as}}(t) = 1 - \exp[-0.2t^{0.5}] = 1 - \exp[-0.2 \cdot (50 \cdot 365.25)^{0.5}] = 1.000$$

Proportional autogenous shrinkage strain over time t:

$$\varepsilon_{\mathrm{ca}}(t) = \beta_{\mathrm{as}}(t) \cdot \varepsilon_{\mathrm{ca},\infty} = 1.000 \cdot 3.750 \cdot 10^{-5} = 3.750 \cdot 10^{-5}$$

Total shrinkage strain:

$$\varepsilon_{\mathrm{cs}} = \varepsilon_{\mathrm{cd}}(t) + \varepsilon_{\mathrm{ca}}(t) = 5.130 \cdot 10^{-4} + 3.750 \cdot 10^{-5} = 5.505 \cdot 10^{-4}$$

Shrinkage curvature:

$$\frac{1}{r_{\mathrm{cs}}} = \varepsilon_{\mathrm{cs}} \cdot n_{\mathrm{eff}} \cdot \left[(1 - \xi) \cdot \frac{S_{\mathrm{I}}}{I_{\mathrm{I}}} + \xi \cdot \frac{S_{\mathrm{II}}}{I_{\mathrm{II}}}\right] = 5.505 \cdot 10^{-4} \cdot 31.584 \cdot \left[(1 - 0.624) \cdot \frac{2.200 \cdot 10^{-5}}{2.558 \cdot 10^{-4}} + 0.624 \cdot \frac{3.855 \cdot 10^{-5}}{1.127 \cdot 10^{-4}}\right]$$
$$= 4.276 \cdot 10^{-3}\ \mathrm{m}^{-1}$$

Finally, deflection due to creep is:

$$\boldsymbol{w_{\mathrm{shrinkage}} = \theta \left[\frac{1}{8} r_{\mathrm{cs}}^{-1} l^2\right] = 1.000 \cdot \left[\frac{1}{8} \cdot 4.276 \cdot 10^{-3} \cdot 3.84^2\right] = 7.881 \cdot 10^{-3}\ \mathrm{m}}$$

6. Literature

[1] EN 1992-1-1:2004+A1:2014. Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings. Brussels: European Committee for Standardization (CEN), 2014.

[2] EN 1990:2002+A1:2005. Eurocode 0: Basis of structural design. Brussels: European Committee for Standardization (CEN), 2005.